THE COLLISIONLESS DECELERATION OF AN IONIZED CLOUD DISPERSING IN A UNIFORM PLASMA IN A MAGNETIC FIELD

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Introduction. The collisionless deceleration of an ionized cloud expanding in a uniform plasma in a magnetic field has been considered in [1-7]. In [1-4] the case was considered where the plasma surrounding the cloud does not significantly affect the motion of the cloud and its deceleration results from interaction with the magnetic field. It was shown that the characteristic deceleration distance R_0 of the cloud by the magnetic field is given by $R_0 = (N_1 m_1 v_0^2 / H_0^2)^{1/3}$, where N_1 is the total number of particles in the cloud, m_1 is the mass on an ion in the cloud, v_0 is a characteristic dispersion velocity, and H_0 is the magnetic field strength. The condition that the effect of the surrounding plasma on the motion of the cloud be small can be obtained as follows. For a strong interaction between the dispersing cloud and the plasma, the characteristic distance R_* over which the cloud decelerates is given by the relation $R_* = (3N_1e_1/4\pi n_*e_2)^{1/3}$, where e_1 and e_2 are the charges of ions in the cloud and plasma, respectively, and n* is the concentration of ions in the surrounding plasma [5]. Obviously the effect of the external plasma can be ignored when $R_* >> R_0$. This condition will be satisfied when the Alfven-Mach number $M_A = v_0/v_A << 1$, where $v_A = H_0/\sqrt{\frac{4\pi n_*m_2}{4\pi n_*m_2}}$ is the Alfven velocity in the external plasma and m_2 is the mass on an ion in the plasma.

When $M_A >> 1$ the dominant effect in the deceleration of the cloud becomes its interaction with the plasma. This problem has been considered in [6, 7] for the case of a cylindrical explosion. In [6] the treatment was based on the Chew-Goldberger-Low equations [8], which apply when the characteristic distance scale of the flow is much larger than the Lamour radius of the ions. In [7] the problem was treated using the one-dimensional hybrid model of [9-11] in which the motion of the ions is treated with the Vlasov equation, and the electron component of the plasma is described as an inertialess fluid. The strength of the interaction between the cloud and the plasma was studied as a function of the ratio $R_{\rm H_1}/R_{\star}$ for $M_{\rm A} >> 1$, where $R_{\rm H_1} = v_0/(e_1H_0/m_1c)$ is the Lamour radius of the ions in the cloud.

In laboratory conditions the cloud of plasma actually disperses practically from a point, so that we have a point explosion and not a cylindrical one. In the deceleration of a cloud from a point explosion, two-dimensional effects can become important because the motion of ions along the magnetic field is completely different from the motion perpendicular to it.

In the present paper we use the two-dimensional hybrid model of [12] to treat the collisionless deceleration of a cloud of plasma from a point explosion. Assuming that the interaction of the cloud with the surrounding dilute plasma comes about through the rotational component of the electric field [7], we obtain the basic deceleration mechanisms and these mechanisms are supported by numerical calculations.

<u>1. Statement of the Problem.</u> The physical model describing a point explosion in a dilute plasma in a magnetic field is the same as the model used in [7] in the solution of the cylindrical explosion problem. In this model the motion of the ions is described by the Vlasov equation.

$$\frac{\partial f_{\alpha}}{\partial t} + \mathbf{v} \frac{\partial f_{\alpha}}{\partial \mathbf{r}} + \frac{e_{\alpha}}{m_{\alpha}} \left(\mathbf{E} + \frac{1}{c} \left[\mathbf{v} \mathbf{H} \right] \right) \frac{\partial f_{\alpha}}{\partial \mathbf{v}} = 0, \qquad (1.1)$$

where f_{α} is the distribution function of ions of type α ; $\alpha = 1$ corresponds to the cloud and $\alpha = 2$ to the surrounding plasma.

The average characteristics are given by

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$$n_{\alpha} = \int f_{\alpha} d\mathbf{v}, \, \mathbf{v}_{\alpha} = \frac{1}{n_{\alpha}} \int f_{\alpha} \mathbf{v} d\mathbf{v}.$$
(1.2)

The equation of motion for the electrons is

$$\mathbf{E} + \frac{1}{c} \left[\mathbf{v}_c \mathbf{H} \right] = 0, \tag{1.3}$$

where v_e is the velocity of the electron "fluid" and E is the electric field. We ignore the inertia and pressure of the electrons and also the loss of electron energy from electron-ion interactions.

The quasineutrality condition of the plasma is assumed to hold:

$$en_e = \sum_{\alpha} e_{\alpha} n_{\alpha}. \tag{1.4}$$

The time evolution of the electric and magnetic fields is described by the Maxwell equations in the quasistationary approximation

$$\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = -\operatorname{rot} \mathbf{E}, \quad \operatorname{rot} \mathbf{H} = \frac{4\pi}{c}\mathbf{j}, \tag{1.5}$$

where j is the current density, which with the help of the quasineutrality condition can be written in the form

$$\mathbf{j} = \sum_{\alpha} e_{\alpha} n_{\alpha} \left(\mathbf{v}_{\alpha} - \mathbf{v}_{e} \right). \tag{1.6}$$

The conditions under which this model is applicable are discussed in [7]. The initial conditions for equations (1.1) through (1.6) are chosen to describe a point explosion into a uniform plasma:

$$f_{1}(\mathbf{r}, \mathbf{v}, t = 0) = N_{1}\delta(\mathbf{r})\psi(v),$$

$$f_{2}(\mathbf{r}, \mathbf{v}, t = 0) = n_{*}\delta(\mathbf{v}), \mathbf{H}(\mathbf{r}, t = 0) = \mathbf{H}_{0},$$
(1.7)

where $v = |\mathbf{v}|$, $\int \psi(v) d\mathbf{v} = 1$, and $\delta(\mathbf{x})$ is the Dirac delta function of vector argument.

2. Interaction of the Expanding Cloud with the Surrounding Medium for Large Alfven-Mach Numbers. We estimate the interaction of the expanding cloud with the plasma using (as in [7]) the conservation of the azimuthal component of the generalized momentum for the charged particles in the axially symmetric magnetic field. In spherical coordinates r, θ , φ with the symmetry axis along $\theta = 0$ this conservation law takes the form

$$r\sin\theta\left(v_{\varphi}+\frac{e_{\alpha}}{m_{\alpha}c}A_{\varphi}\right)=r_{0}\sin\theta_{0}\left(v_{\varphi_{0}}+\frac{e_{\alpha}}{m_{\alpha}c}A_{\varphi_{0}}\right),$$
(2.1)

where quantities with subscript 0 refer to the initial instant of time. A_{φ} is the azimuthal component of the vector potential of the magnetic field: H = rot A.

From (1.3) through (1.6) the following equation is obtained for A:

$$\frac{\partial \mathbf{A}}{\partial t} = \left[\frac{1}{en_e} \sum_{\alpha} e_{\alpha} n_{\alpha} \mathbf{v}_{\alpha} - \frac{c}{4\pi en_e} \operatorname{rot} \operatorname{rot} \mathbf{A}, \operatorname{rot} \mathbf{A}\right].$$
(2.2)

Let the spatial scale over which the magnetic field changes be R_{*}, then the ratio of the term $\frac{c}{4\pi n_e e}$ rotrot A in (2.2) to the term $\frac{1}{en_e}\sum_{\alpha}e_{\alpha}n_{\alpha}v_{\alpha}$ will be proportional to M_A^{-2} and when $M_A >> 1$ we can use in place of (2.2) the approximate equation

$$\frac{\partial \mathbf{A}}{\partial t} = \frac{1}{e n_e} \left[\sum_{\alpha} e_{\alpha} n_{\alpha} \mathbf{v}_{\alpha}, \operatorname{rot} \mathbf{A} \right].$$
(2.3)

When the interaction between the cloud and surrounding plasma is weak, the Vlasov equations for the ions and equation (2.3) can be solved by successive approximation.

Ignoring the effect of the fields on the motion of the ions and using the initial con-

ditions on the distribution functions f_1 , f_2 as given by (1.7) we have

$$f_1(\mathbf{r}, \mathbf{v}, t) = N_1 \delta(\mathbf{r} - \mathbf{v}t) \psi\left(\frac{|\mathbf{r}|}{t}\right), \qquad (2.4)$$

$$f_2(\mathbf{r}, \mathbf{v}, t) = n_* \delta(\mathbf{v}).$$

The average characteristics of the plasma corresponding to these distribution functions are, in spherical coordinates

$$n_{1} = N_{1}t^{-3}\psi\left(\frac{r}{t}\right), \quad v_{1r} = \frac{r}{t}, \quad v_{1\varphi} = v_{1\theta} = 0,$$

$$n_{2} = n_{*}, \quad v_{2r} = v_{2\theta} = v_{2\varphi} = 0.$$
(2.5)

Since n_{α} , $v_{\alpha r}$ do not depend on θ , ϕ , and $v_{\alpha \theta} = v_{\alpha \phi} = 0$, (2.3) reduces to a single equation for the only nonzero component of the vector potential

$$A_{\varphi} = A(r, t) \sin \theta.$$

Using (2.5) we obtain the following equation for A(r, t)

$$\frac{\partial A}{\partial t} + \frac{\frac{e_1}{e_2} \frac{N_1}{n_{\bullet}} \psi\left(\frac{r}{t}\right) \frac{r}{t^4}}{1 + \frac{e_1}{e_2} \frac{N_1}{n_{\bullet}} \psi\left(\frac{r}{t}\right) \frac{1}{t^3}} \frac{1}{r} \frac{\partial}{\partial r} (Ar) = 0.$$
(2.6)

For arbitrary $\psi(r/t)$ the solution of (2.6) satisfying the condition A(r, t) \rightarrow H₀r/2 at $r \rightarrow \infty$ is

$$A(r, t) = \begin{cases} 0, & 0 \leq r \leq r^*, \\ \frac{H_0 R_*^2}{2r} \left\{ \left(\frac{r}{R_*}\right)^3 + 4\pi \int_0^{r/t} \psi(v) v^2 dv - 1 \right\}^{2/3}, & r^* \leq r. \\ R_* = (3e_1 N_1 / 4\pi n_* e_2)^{1/3}, \end{cases}$$
(2.7)

and r* satisfies the equation

$$\left(\frac{r^{*}}{R_{*}}\right)^{3} + 4\pi \int_{0}^{r^{*}/t} \psi(v) v^{2} dv - 1 = 0.$$

A physical interpretation of this solution is given in [7].

Knowing the azimuthal component A_{φ} of the vector potential, we can estimate the energy transferred from the dispersing cloud to the ions of the medium. In the approximation considered here, only the azimuthal component of the electric field E_{φ} will be nonzero for r* $\leq r < \infty$. Assuming as in [7] that the ions of the medium suffer only small displacements in the r and θ directions from the initial position r₀, θ_0 , and using (2.1) we obtain

$$v_{20} = (e_2/cm_2)(H_0 r/2 - A(r, t)) \sin \theta, \qquad (2.8)$$

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where A(r, t) is given by (2.7).

Then the total energy W_2 transferred from the cloud at time t is given by

$$W_2 = m_2 n_* \int_0^{\pi} 2\pi \sin \theta d\theta \int_0^{\infty} r^2 dr \, \frac{v_{2\varphi}^2}{2}.$$

Using the above expression for $\dot{\nu}_{2\phi}$ we obtain

$$W_{2} = \frac{\pi}{3} n_{*} \frac{e_{2}^{2}}{c^{2}m_{2}} H_{0}^{2} R_{*}^{5} \left\{ \frac{1}{5} \left(\frac{r^{*}}{R_{*}} \right)^{5} + \int_{r^{*}/R_{*}}^{\infty} \left[x^{2} - \left(x^{3} + 4\pi \int_{0}^{\frac{R_{*}x}{t}} \psi(v) v^{2} dv - 1 \right)^{2/3} \right]^{2} dx \right\}.$$
(2.9)

It is easily seen that in the limit $t \rightarrow \infty W_2$ approaches the following limit independent of the function $\psi(v)$:

$$W_{2\infty} = \frac{1}{3} \pi n_* \frac{e_2^2}{c^2 m_2} H_0^2 R_*^5 \left\{ \frac{1}{5} + \int_1^\infty \left[x^2 - (x^3 - 1)^{2/3} \right]^2 dx \right\}.$$
 (2.10)

Since the total initial energy of the cloud particles is $W_0 = N_1 m_1 2\pi \int \psi(v) v^4 dv$ we obtain the following result for the fraction of energy transferred from the cloud to the medium:

$$\frac{W_{2\infty}}{W_0} = \frac{1}{2} \frac{R_*^2}{r_{H_1} r_{H_2}} I = \frac{1}{2} I \delta,$$
(2.11)

where $r_{H_1} = v_1/(e_1H_0/m_1c)$, $r_{H_2} = v_1/(e_2H_0/m_2c)$ are the Lamour radii for ions in the cloud and medium, respectively, $v_1 = (2W_0/N_1m_1)^{1/2}$; and finally

$$I = \frac{1}{5} + \int_{1}^{\infty} \left[x^2 - (x^3 - 1)^{2/3} \right]^2 dx \simeq 0.71.$$

Thus the final fraction of energy transferred does not depend on the details of the initial velocity distribution of cloud particles, but is determined only by the average characteristics of the cloud and surrounding plasma.

The above results are generally correct only when $W_{2\,\infty}/W_0 \ll 1$, i.e., when the interaction is weak. However (2.11) can be used to estimate the value of δ for which the interaction becomes strong. Putting $W_{2\infty}/W_0 = 1$ we find $\delta = 2/I \simeq 2.8$. It is of interest to estimate the fraction of energy transferred to the medium in the interval of angles (θ , θ + d θ). This will be given by the ratio $W_{2\,\Theta}/W_{1\,\Theta}$ where

$$W_{1\theta} = N_1 m_1 \pi \sin \theta d\theta \int_0^\infty \psi(v) v^4 dv;$$

 $W_{2\theta} = m_2 n_* \pi \sin \theta d\theta \int_0^\infty r^2 dr v_{2\varphi}^2.$

Using the above expression for $v_{2 p}$ and the definition of the parameter δ we have

$$W_{2\theta}/W_{1\theta} = (3/4)\delta I \sin^2 \theta_{\bullet}$$
(2.12)

From the condition $W_{2\theta} \simeq W_{1\theta}$ we can find the angle θ_{\star} dividing the regions of strong and weak interactions:

$$\theta_* = \arcsin\left(\frac{4}{3\delta I}\right)^{1/2}.$$
 (2.13)

For $\theta \ge \theta_*$ the interaction of the cloud and the surrounding plasma leads to practically a total transfer of energy from cloud to medium; for $\theta < \theta_*$ the medium is weakly perturbed and does not affect the motion of the cloud particles.

<u>3. Results of the Calculations.</u> In the numerical solution of (1.1) through (1.6) (the hybrid model with initial conditions corresponding ot a point explosion) we used the following units: v_0 , the maximum dispersion velocity of the cloud particles; H_0 , the unperturbed magnetic field strength; $R_{H_2} = v_0/(e_2H_0/m_2c)$, the Lamour radius of ions in the surrounding plasma in the unperturbed field; n*, the concentration of ions in the surrounding plasma, m_2 , e_2 , their mass and charge. The function $\psi(v)$ was chosen to be

$$\psi(v) = \frac{3}{4\pi v_0^3} \theta(v_0 - v),$$

where

$$\theta(x) = \begin{cases} 1, & x \ge 0, \\ 0, & x < 0. \end{cases}$$

In these units the initial distribution functions are given by











Fig. 5









Fig. 6

$$f_{1}(\mathbf{r}, \mathbf{v}, t=0) = \left(\frac{3}{5}\right)^{3/2} \delta^{3/2} \left(\frac{e_{2}}{e_{1}}\right)^{5/2} \left(\frac{m_{1}}{m_{2}}\right)^{3/2} \frac{v_{r}^{2}}{r^{2}} \delta(r) \,\delta(v_{\theta}) \quad \delta(v_{\phi}) \,\theta(1-v_{r}) \,\theta(v_{r}),$$

$$f_2(\mathbf{r}, \mathbf{v}, t=0) = \delta(v_r)\delta(v_{\theta})\delta(v_{\phi}).$$

We ran two sets of calculations for parameters given by $m_1/m_2 = 1$, $e_1/e_2 = 1$, $\delta = 4.2$ and $m_1/m_2 = 2$, $e_1/e_2 = 1$, $\delta = 1.33$. The two sets correspond to explosions of the same energy and same amount of dispersing material. The surrounding media are also identical, and the only difference is that in the second set the cloud particles are twice the mass of those in the first set. We used the value $M_A = 10$ for both sets of calculations.

The results are shown in Figs. 1-6. For ease in interpreting the graphs the quantity

 $R_m = (3N_1m_1/4\pi n_*m_2)^{1/3}$

was used as a scale of length. In Fig. 1 we show the time dependence of the energy of the dispersing cloud obtained numerically (solid curves) and with the use of (2.9) (dashed curves); curves 1 and 2 refer to the case $m_1/m_2 = 2$, curves 3 and 4 to $m_1/m_2 = 1$. We see the close correspondence between the calculated and approximate curves for the case m_1/m_2 for practically all values of the time; for the case m_1/m_2 1 the agreement is good only at small times when the fraction of energy transferred is small: $(W_0 - W_1)/W_0 \leq 0.5$.

In Fig. 2 we show the dependence on θ of the energy transferred to the surrounding plasma per unit solid angle at time t = $2R_m/v_0$. The solid curves give the calculated dependence, the dashed curves show the dependence

$$\frac{W_{2\theta}}{W_{1\theta}} = \frac{W_{2\theta}}{W_{1\theta}} \left(\theta = \frac{\pi}{2}\right) \sin^2 \theta,$$

from (2.12). Curves 1 and 2 refer to $m_1/m_2 = 2$; curves 3 and 4 to $m_1/m_2 = 1$. The close correspondence between the numerical calculation and the approximate theory can be seen.

In Fig. 3 the lines of force of the magnetic field in the plane $\varphi = \text{const}$ are shown for $m_1/m_2 = 1$ at time $t = 4R_m/v_0$. In this plane the coordinates z and ρ are given in terms of r, θ by the relations

$$z = r \cos \theta$$
, $\rho = r \sin \theta$.

From Fig. 3 it is seen that the region where the magnetic field is squeezed out is, to a good approximation, spherically symmetric. The compression of the field (increase in density of field lines) is maximum at the equator ($\theta = \pi/2$) and decreases as we approach the poles ($\theta = 0, \pi$). The dashed line in Fig. 3 shows the boundary of the region (in the plane $\varphi =$ const) occupied by cloud particles. This region has a remarkable shape. In the interval of angles $\theta_0 \leq \theta \leq \pi - \theta_0$ the boundary is a section of a sphere; in the angular intervals $0 \leq \theta \leq \theta_0$ and $\pi - \theta_0 \leq \theta \leq \pi$ the boundary is strongly drawn out along the magnetic field lines. The size of the region in this direction will be determined by the freely moving particles and will be proportional to t. The maximum size of the region occupied by particles in the direction transverse to the external magnetic field is determined by the Lamour radius of the particles initially moving perpendicular to the field. In Fig. 3 the angle θ_* dividing the regions of strong and weak interactions is also shown; its value is calculated from (2.13). The agreement between θ_* and θ_0 is seen to be satisfactory.

In Figs. 4-6 the dependence of the components H_r , H_θ , H_φ on r for different values of θ is shown. The curves are drawn for $m_1/m_2 = 1$, and time $t = R_m/v_0$. As expected, the maximum perturbations H_r and H_θ are observed near $\theta = \pi/2$. The maximum value of the magnetic field component H_φ is reached near $\theta = \pi/4$; this is because $H_\varphi = 0$ for the unperturbed field and in the first approximation H_φ is generated by the components H_r , H_θ calculated with the vector potential A_φ of (2.7). Since $H_r \sim \cos \theta$, $H_\theta \sim \sin \theta$ we see that $H_\varphi \sim \cos \theta \sin \theta$.

Our results lead to the following conclusions. For a point explosion into a dilute plasma in a magnetic field, if the Alfven-Mach number M_A is large, the strength of the interaction between the dispersing cloud and the surrounding plasma is determined by the parameter $\delta = R_*^2/R_{H_1}R_{H_2}$. In the interval of θ given by $\theta \left(\theta_* \leqslant \theta \leqslant \pi - \theta_* \right)^{1/2}$, the interaction is strongest and the plasma flow is spherically symmetric so that two-dimensional effects are unimportant. Outside this interval of angles, the ions of the dispersing cloud move in an almost unperturbed magnetic field with practically no loss of energy.

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CHARACTERISTICS OF THE STEADY-STATE FLOW OF A TWO-TEMPERATURE

ARGON-ARC PLASMA IN A CHANNEL

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Design of high-temperature gas heaters with given characteristics is hindered by the lack of applicable models closely adapted to real conditions. This applies particularly to electric-arc plasma sources that produce thermally nonequilibrium plasma. Theoretical studies have been made [1-6] of the arc characteristics in the two-temperature approximation for the initial and steady-state parts. However, most of these are restricted to arc burning in the absence of a flow [1, 3] and laminar gas flow in the discharge channel [2, 4, 5], while in [4] the calculations were performed with a very crude approximation in order to obtain a simple solution. The most accurate analysis is to be found in [2, 5], while in [6] there is an estimate of the effects from turbulence. No detailed analysis has been made of the effects of the transition from laminar flow to turbulent on the characteristics of a twotemperature flow of electric-arc plasma. Also, the properties of the plasma have been derived from formulas that are not always reliable, which substantially hinders examination of the accuracy and applicability of the results.

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We have examined the effects of laminar and turbulent flow on the characteristics of a two-temperature argon plasma in the steady-state part of an electric arc in a cylindrical channel. We have examined the existing formulas for the plasma properties and have selected those that agree best with experiment.

It has been found that the following system of equations can be used to describe the phenomena occurring in an electric arc stabilized by the walls of a cylindrical channel and bearing a longitudinal gas flow:

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